The inclusive $B \to \eta' X_s$ decay and $b \to sg^*$ form factors

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Abstract

We compute the branching ratio of inclusive $B \to \eta' X_s$ decay based upon the QCD anomaly mechanism: $b \to s + g^* \to s + g + \eta'$. To obtain a reliable $B \to \eta' X_s$ branching ratio, we calculate the $b \to s + g^*$ form factors up to the next-to-leading-logarithmic(NLL) approximation. We point out that the Standard Model prediction for $B \to \eta' X_s$ is consistent with the CLEO data, in contrast to the claims of some previous works.

1. Introduction

The observations of exclusive $B \to \eta' K$[1] and inclusive $B \to \eta' X_s$[2] decays with high momentum η' mesons have stimulated many theoretical activities [3, 4, 5, 6, 7, 8, 9, 10]. The experimental fitting[2] shows that the dominant mechanism for the inclusive mode should be $b \to sg^* \to sg\eta'$[3, 4] where the η' meson is produced via the anomalous $\eta' - g - g$ coupling. According to a previous analysis[4], this mechanism within the Standard Model(SM) can only account for 1/3 of the measured branching ratio: $\mathcal{B}(B \to \eta' X_s) = \left[6.2 \pm 1.6(\text{stat}) \pm 1.3(\text{syst})^{+0.0}_{-1.5}(\text{bkg})\right] \times 10^{-4}$[2] with $2.0 < p_{\eta'} < 2.7$ GeV. Furthermore, the subleading mechanism for $B \to \eta' X_s$, based upon four-quark operators of the effective weak Hamiltonian[5, 6], is not sufficient to make up the above deficiency. Due to results of these analyses, the possibility of an enhanced $b \to sg$ or other mechanisms arising from physics beyond the Standard Model are raised to account for the above discrepancy in branching ratios[4, 6, 7]. In order to see if new physics should play any role in $B \to \eta' X_s$, one has to have a better understanding on the SM prediction. In this talk, we present a careful analysis on $B \to \eta' X_s$[11] using the next-to-leading order effective Hamiltonian. In section 2, we illustrate how to compute the off-shell $b \to sg^*$ form factors in such a framework. In particular, the QCD equation of motion is applied to transform the charge-radius form factor of $b \to sg^*$ into the structures of certain four-quark operators. Therefore the effective weak Hamiltonian is shown applicable for computing such a form factor. In section 3, we calculate the branching ratio and the recoil spectrum of $B \to \eta' X_s$ decay. The results are found to be consistent with the CLEO measurement[2]. Section 4 is the conclusion.

† based upon a work done with X.-G. He

2. QCD equation of motion and $b \to s + g^*$ form factors

The effective Hamiltonian[12] relevant to the $B \to \eta' X_s$ decay is given by:

$$H_{eff} = -\frac{G_F}{\sqrt{2}} V_{ts}^* V_{tb} \left(\sum_{i=1}^{6} C_i(\mu) O_i(\mu) + C_8(\mu) O_8(\mu)\right), \tag{1}$$

with

$$\begin{aligned} O_1 &= (\bar{s}_i c_j)_{V-A} (\bar{c}_j b_i)_{V-A}, \\ O_2 &= (\bar{s}_i c_i)_{V-A} (\bar{c}_j b_j)_{V-A}, \\ O_{3,5} &= (\bar{s}_i b_i)_{V-A} \sum_q (\bar{q}_j q_j)_{V \mp A}, \\ O_{4,6} &= (\bar{s}_i b_j)_{V-A} \sum_q (\bar{q}_j q_i)_{V \mp A}, \\ O_8 &= -\frac{g_s m_b}{4\pi^2} \bar{s}_i \sigma^{\mu\nu} P_R T^a_{ij} b_j G^a_{\mu\nu}, \end{aligned} \tag{2}$$

where $V \pm A \equiv 1 \pm \gamma_5$. Precisely speaking, this effective Hamiltonian can be used to calculate the off-shell $b \to sg^*$ form factors which are expressed as

$$\begin{aligned} \Gamma_\mu^{bsg} = &- \frac{G_F}{\sqrt{2}} V_{ts}^* V_{tb} \frac{g_s}{4\pi^2} (F_1 \bar{s}(q^2 \gamma_\mu - \not{q}\, q_\mu) L T^a b \\ &- i F_2 m_b \bar{s} \sigma_{\mu\nu} q^\nu R T^a b). \end{aligned} \tag{3}$$

It is easily seen that $F_2 = -2C_8(\mu)$. However, the connection between F_1 and the effective Hamiltonian H_{eff} is less obvious. One may acquire some hints by rearranging the QCD penguin operators:

$$\sum_{i=3}^{6} C_i O_i = 2(C_4 + C_6)O_V - 2(C_4 - C_6)O_A + (C_3 + \frac{C_4}{N_c})O_3 + (C_5 + \frac{C_6}{N_c})O_5, \quad (4)$$

where

$$O_A = \bar{s}\gamma_\mu(1-\gamma_5)T^a b \sum_q \bar{q}\gamma^\mu\gamma_5 T^a q,$$
$$O_V = \bar{s}\gamma_\mu(1-\gamma_5)T^a b \sum_q \bar{q}\gamma^\mu T^a q. \quad (5)$$

Since the light-quark bilinear in O_V carries the quantum number of a gluon, one expects[3] O_V give contributions to the $b \to sg^*$ form factors. In fact, by applying the QCD equation of motion, $D_\nu G_a^{\mu\nu} = g_s \sum \bar{q}\gamma^\mu T^a q$, we have $O_V = (1/g_s)\bar{s}\gamma_\mu(1-\gamma_5)T^a b D_\nu G_a^{\mu\nu}$. In this form, O_V clearly contributes to the charge-radius form factor F_1. Let us denote this part of contribution as F_1^a. We have $F_1^a = 4\pi/\alpha_s \cdot (C_4(\mu) + C_6(\mu))$. We remark that, at the NLL level, F_1 should also receive contributions from one-loop matrix elements. The dominant contribution, denoted as F_1^b, arises from the operator O_2 where its charm-quark-pair meets to form a gluon. In the NDR scheme, we find $F_1^b = 4\pi/\alpha_s \cdot (\bar{C}_4(q^2,\mu) + \bar{C}_6(q^2,\mu))$ where q^2 is the invariant mass of the gluon and

$$\bar{C}_4(q^2,\mu) = \bar{C}_6(q^2,\mu) = \frac{\alpha_s(\mu)}{8\pi} C_2(\mu)\left(\frac{2}{3} + G(m_c^2, q^2, \mu)\right), \quad (6)$$

with[13]

$$G(m_c^2, q^2, \mu) = 4\int_0^1 x(1-x)dx \times \log\left(\frac{m_c^2 - x(1-x)q^2}{\mu^2}\right). \quad (7)$$

We point out that F_1^b, the O_2 contribution, is not negligible. For $\mu = 5$ GeV, one has $F_1^a = -4.03$ and $\mathrm{Re}(F_1^b)$(the real part of F_1^b) ranging between -1.5 and -3 for $0.2 < q^2/m_b^2 < 0.7$. The peak value $\mathrm{Re}(F_1^b) \equiv -3$ occurs at the charm-pair threshold $q^2 = 4m_c^2$. For $q^2 > 4m_c^2$, F_1^b develops an imaginary part whose value is roughly $2i$. Concerning previous results on F_1, we note that Ref. [3] intends to compue F_1 with effective weak Hamiltonian. However, only the contribution by F_1^a is considered. Ref. [4] took $F_1 = -5$ which is a result of an one-loop calculation[15]. Clearly the q^2 dependencies of F_1 are also absent. As we shall see in the next section, the contribution by F_1^b, which is not included in previous works, can significantly enhance the $B \to \eta' X_s$ branching ratio such that SM prediction is consistent with the CLEO measurement.

3. The inclusive $B \to \eta' X_s$ decay

In this decay, the η' final state arises from the off-shell gluon splitting, $g^* \to g\eta'$, where g^* is produced through $b \to sg^*$. The branching-ratio distribution of $b(p) \to s(p') + g(k) + \eta'(k')$ is[4]:

$$\frac{d^2\mathcal{B}(b \to sg\eta')}{dxdy} \cong 0.2\cos^2\theta\left(\frac{g_s(\mu)}{4\pi^2}\right)^2 \frac{a_g^2(\mu)m_b^2}{4} \times \left(|\Delta F_1|^2 c_0 + \mathrm{Re}(\Delta F_1 F_2^*)\frac{c_1}{y} + |\Delta F_2|^2 \frac{c_2}{y^2}\right), \quad (8)$$

where $a_g(\mu) \equiv \sqrt{N_F}\alpha_s(\mu)/\pi f_{\eta'}$ is the strength of $\eta' - g - g$ vertex: $a_g \cos\theta \epsilon_{\mu\nu\alpha\beta} q^\alpha k^\beta$ with q and k the momenta of two gluons; $x \equiv (p'+k)^2/m_b^2$ and $y \equiv (k+k')^2/m_b^2$; c_0, c_1 and c_2 are functions of x and y given by:

$$c_0 = \left[-x^2 y + (1-y)(y-x')(x + \frac{y}{2} - \frac{x'}{2})\right],$$
$$c_2 = \left[x^2 y^2 - (1-y)(y-x')(xy - \frac{y}{2} + \frac{x'}{2})\right],$$
$$c_1 = (1-y)(y-x')^2, \quad (9)$$

with $x' \equiv m_{\eta'}^2/m_b^2$; and the $\eta' - \eta$ mixing angle θ is taken to be -15.4^o[16]. Following the approach in Ref. [4], we evaluate the $\alpha_s(\mu)$ in a_g at the running scale $\mu^2 = q^2$. Taking $\mu = 5$ GeV for evaluating other scale-dependent quantities, we find $\mathcal{B}(b \to sg\eta') = 6.4 \times 10^{-4}$ with the cut $m_X \equiv \sqrt{(k+p')^2} \leq 2.35$ GeV imposed in the CLEO measurement[2]. This branching ratio is consistent with CLEO's measurement on $\mathcal{B}(B \to \eta' X_s)$ branching ratio[2]. Without the kinematic cut, we obtain $\mathcal{B}(b \to sg\eta') = 1.2 \times 10^{-3}$, which is much larger than 4.3×10^{-4} calculated previously[4]. Clearly this enhancement is due to the contribution of F_1^b, which increases the magnitude of F_1 and thus enhances the the branching ratio of $b \to sg\eta'$ according to Eq. (8). Since Ref.[3] also neglects the contribution by F_1^b, its prediction on $\mathcal{B}(b \to sg\eta')$

would be much smaller than ours if the $\eta' - g - g$ coupling there is also evaluated at the running scale $\mu^2 = q^2$. However, the prediction by Ref. [3] is comparable to ours, since, in that work, the $\alpha_s(\mu)$ in a_g is evaluated at the lowest possible scale $\mu^2 = m^2_{\eta'}$, and the interference between the contributions by F_1 and F_2 is constructive rather than destructive reported here and in Ref.[4].

To ascertain our calculation, we also check the μ dependence of the $b \to sg\eta'$ branching ratio. Using NDR scheme with $\mu = 2.5$ GeV and imposing the kinematic cut $m_X \leq 2.35$ GeV, we find $\mathcal{B}(b \to sg\eta') = 7.1 \times 10^{-4}$, which is only 10% larger than the branching ratio obtained at $\mu = 5$ GeV. This insensitivity to the scale-choice is reassuring. We also obtain the spectrum $d\mathcal{B}(b \to sg\eta')/dm_X$ which has been shown in Ref.[11] and will not be displayed here. The peak of the spectrum corresponds to $m_X \approx 2.4$ GeV. As pointed out in Ref. [2], this type of spectrum gives the best fit to the CLEO data.

4. Concluding remarks

We have calculated the branching ratio of $b \to sg\eta'$ by including the NLL correction to the $b \to sg^*$ form factors. By evaluating the $\eta' - g - g$ coupling at the running scale $\mu = q^2$ and cutting the recoil-mass m_X at 2.35 GeV, we obtained $\mathcal{B}(b \to sg\eta') = (6.4 - 7.1) \times 10^{-4}$ depending on the choice of the renormalization scale for evaluating the $b \to sg^*$ form factors. We have not addressed the possible form-factor suppression of the $\eta' - g - g$ vertex, which occurs as the gluons attached to the vertex go farther off-shell[3, 4, 6]. So far it remains unclear how much the form-factor suppression might be. However, comparing our prediction with the CLEO measurement on $\mathcal{B}(B \to \eta' X_s)$, which still has a large error bar, it remains possible that the anomaly-induced process $b \to sg\eta'$ could account for the CLEO data within the framework of the Standard Model.

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